## CXXXIX.-Molecular Volumes at Absolute Zero. Part III. Zero Volumes, Parachors, and Molecular Diameters.

By Samuel Sugden.
In Parts I and II of this series (J., 1927, 1780, 1786) no reference was made to the work of Biltz in the same field (Nachr. Ges. Wiss. Göttingen, 1926, 45; Annalen, 1927, 453, 259). The first of these communications appeared in a journal not readily accessible in this country, and the second, which deals with additive relations in organic compounds, was published only a few months before the author's work was submitted for publication.

In these papers, Biltz has extended to organic substances the studies of limiting molecular volumes in the liquid and the solid state on which he has been engaged for some years. His zero volumes are obtained from a variety of different sources, viz., extrapolation of liquid densities, densities in the solid state, and limiting molecular volumes deduced from the $X$-ray analysis of crystals; the values obtained by these methods usually agree within a few units \%. For compounds, Biltz finds an additive relationship of the form $V_{0}=\Sigma\left(n v_{0} / m\right)$, in which $V_{0}$ is the zero volume of the compound, and $v_{0}$, the zero volume of any one constituent free element obtained directly from its density, etc., is multiplied by the appropriate coefficient $n / m$, where $n$ and $m$ are small whole numbers. In many cases $n / m=1$, e.g., for carbon and the halogens; for hydrogen $n / m=\frac{1}{2}$, i.e., this element in its compounds is assigned half the molecular volume it is found to occupy in the elementary condition. Similarly, for carbonyl oxygen $n / m=1$, and for hydroxylic oxygen $n / m=\frac{1}{3}$.

This method of treating zero volumes differs in many respects from that adopted by the author (loc. cit.). Nevertheless, the atomic and structural constants deduced by the two methods show a
fairly close agreement, as may be seen from the examples shown below.

|  | $\begin{gathered} V_{0} \\ \text { (Biltz) } . \end{gathered}$ | $\underset{\text { (Sugden). }}{V_{0}}$ |  | $\begin{gathered} V_{0} \\ (\text { Biltz). } \end{gathered}$ | $\underset{\text { (Sugden). }}{V_{0}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| C | $3 \cdot 4$ | $1 \cdot 1$ | O (carbonyl)..... | $10 \cdot 7$ | 13.0 |
| H | $5 \cdot 8$ | 6.7 | O (hydroxyl) ... | $3 \cdot 6$ | $3 \cdot 0$ |
| $\mathrm{CH}_{2}$ | 15.0 | 14.5 | 0 (ethers) | $3 \cdot 6$ | $5 \cdot 0$ |
| Cl | 16.5 | $19 \cdot 3$ | N (amines) | $0 \cdot 0$ | $0 \cdot 0$ |
| $\stackrel{\mathrm{Br}}{ }$ | 19.5 | $22 \cdot 1$ | Double bond ... | $4 \cdot 0$ | $8 \cdot 0$ |
| I | $24 \cdot 3$ | 28.3 | Triple bond ...... | 12.0 | 15.5 |

Furthermore, the zero volumes of compounds deduced from the densities of liquids or solids, or from crystal analysis, have been shown by Biltz to agree closely, and can be predicted with a fair degree of accuracy by using either series of constants.

It is therefore of interest to examine the relationship between zero volumes (and, in addition, parachors and critical volumes) and the actual dimensions of the molecules. The latter quantity is somewhat difficult to define with certainty, but probably the best measure of the diameter of a molecule is provided by the modern work on the viscosity of gases. The mathematical theory of Chapman (Phil. Trans., 1916, $A, 216,347$ ) leads to the formula

$$
\begin{equation*}
\pi \sigma^{2}=\frac{0 \cdot 491(1+\varepsilon) m \bar{V}}{\sqrt{2} \eta(1+C / T)} . \quad . \quad . \tag{1}
\end{equation*}
$$

Here $\eta$ is the viscosity of the gas at an absolute temperature $T$, $m$ is the mass of the molecule, $\sigma$ its diameter, and $\bar{V}$ its mean velocity (which varies as $\sqrt{T}$ ); $\varepsilon$ is usually very small and can be neglected, and $C$ is Sutherland's constant. Provided that Sutherland's law (Phil. Mag., 1893, 36, 507) holds, this formula gives a value for $\sigma$ which is independent of temperature.

It will be seen from the formula that $\eta$ is inversely proportional to the cross-sectional area of the molecule, and it is undoubtedly better to follow the procedure of Rankine (ibid., 1921, 42, 601, 615) and discuss collision areas rather than molecular diameters. In the present problem, however, molecular volumes have to be compared with the viscosity data, so some hypothesis about the shape of the molecule has to be made. In Table I, the molecules are regarded as spheres and the diameters of these spheres are compared. For the inert gases this procedure is probably accurate, but some uncertainty is introduced into the values for more complex molecules.

The volume occupied by an assemblage of $N$ closely packed spheres of diameter $\sigma$ is $V=N \sigma^{3} / \sqrt{2}$, hence $\sigma=\sqrt[3]{\sqrt{2} V / N}=1.326 \times$ $10^{-8} \sqrt[3]{V} \mathrm{~cm}$., if $V$ is the volume (in c.c.) of 1 g.-mol. In the liquid state we may assume a random distribution of molecules, so that the value of $\sigma$ obtained in this way gives the average distance between the centres of adjacent molecules. In Table I the column headed $\sigma_{n}$
gives the molecular diameter (in Ångström units) calculated from the gas viscosity by formula (1). The data for the last five substances are taken from Hertzfeld ("Handbuch der Physik," Springer, Berlin, 1926, 22, 409); for the others the values of $\sigma_{\eta}$ are calculated from the observations of Rankine (loc.cit.). The columns headed $\sigma_{0}, \sigma_{p}$, and $\sigma_{c}$ give the mean distances between the molecules respectively at absolute zero, as calculated from the author's zero volumes, at unit surface tension, as calculated from the parachor, and at the critical temperature. The last three columns give the ratio of these mean distances to $\sigma_{\eta}$.

If the molecules of a supercooled liquid are in contact with one another at absolute zero, then $\sigma_{0} / \sigma_{\eta}$ should be approximately unity. There are, however, many reasons for supposing that the field of force around a molecule, which at small distances produces attraction, changes to one giving a force of repulsion when two molecules approach each other very closely. When thermal agitation ceases at $0^{\circ}$ Abs., the molecules are probably distributed at such distances that there is neither attraction nor repulsion between them. On the other hand, the value of $\sigma_{\eta}$ gives the closest approach of two rapidly moving molecules in a gaseous collision; hence $\sigma_{0} / \sigma_{\eta}$ would be expected to be rather greater than unity. It will be seen that most of the substances give a value for this ratio of $1.2-1.3$. Hydrogen and helium give much higher values, but this is only to be expected since the van der Waals attractive force for these

Table I.

| Substance | $V_{0}$. | [P]. | $V_{c}$. | $\sigma_{\eta}$. | $\sigma_{0}$. | $\sigma_{p}$. | $\sigma_{c}$. | $\sigma_{0} / \sigma_{\eta}$. | $\sigma_{p} / \sigma_{\eta}$. | $\sigma_{c} / \sigma_{\eta}$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| He | $22 \cdot 3$ | $20 \cdot 2$ | $57 \cdot 6$ | 1.94 | $3 \cdot 73$ | $3 \cdot 61$ | $5 \cdot 12$ | 1.92 | 1.86 | $2 \cdot 64$ |
| Ne | $12 \cdot 8$ | $25 \cdot 1$ | 41.8 | $2 \cdot 32$ | $3 \cdot 10$ | $3 \cdot 88$ | $4 \cdot 60$ | $1 \cdot 34$ | 1.67 | 1.98 |
| A | 22-4 | $53 \cdot 8$ | $75 \cdot 2$ | $2 \cdot 90$ | $3 \cdot 74$ | $5 \cdot 01$ | $5 \cdot 73$ | $1 \cdot 29$ | 1.73 | 1.97 |
| Xe | $33 \cdot 0$ |  | 112.7 | $3 \cdot 44$ | $4 \cdot 25$ |  | 6.41 | $1 \cdot 24$ |  | 1.86 |
| $\mathrm{H}_{2}$ | $22 \cdot 0$ | $35 \cdot 1$ | 46.9 | $2 \cdot 18$ | $3 \cdot 72$ | $4 \cdot 34$ | $4 \cdot 78$ | 1.71 | 1.99 | $2 \cdot 19$ |
| $\mathrm{Cl}_{2}$ | $38 \cdot 6$ | 111.5 | 123.7 | $4 \cdot 12$ | $4 \cdot 48$ | $6 \cdot 38$ | 6.61 | 1.09 | 1.55 | $1 \cdot 60$ |
| $\mathrm{Br}_{2}$ | $44 \cdot 2$ | $132 \cdot 1$ | 135.7 | $4 \cdot 04$ | $4 \cdot 69$ | 6.75 | 6.81 | $1 \cdot 16$ | 1.67 | 1.69 |
| HCl | $26 \cdot 0$ | $67 \cdot 8$ | 89.5 | 2.93 | 3.93 | $5 \cdot 41$ | $5 \cdot 93$ | $1 \cdot 34$ | 1.85 | $2 \cdot 02$ |
| HBr | 28.8 | $85 \cdot 4$ |  | $3 \cdot 12$ | $4 \cdot 07$ | $5 \cdot 84$ | - | $1 \cdot 30$ | 1.87 |  |
| HI | $35 \cdot 0$ | $105 \cdot 3$ |  | $3 \cdot 43$ | $4 \cdot 34$ | $6 \cdot 26$ |  | $1 \cdot 26$ | 1.83 |  |
| $\mathrm{NH}_{3}$ | 19.1 | $63 \cdot 8$ | 71.9 | $2 \cdot 85$ | $3 \cdot 63$ | $5 \cdot 30$ | $5 \cdot 51$ | 1.27 | 1.86 | 1.95 |
| $\mathrm{C}_{2} \mathrm{H}_{4}$ | $37 \cdot 4$ | 99.5 |  | $3 \cdot 71$ | $4 \cdot 43$ | 6.14 |  | 1.20 | 1.66 |  |
| $\mathrm{C}_{6} \mathrm{H}_{6}$Ethyl $\ldots$ |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| iso. |  |  |  |  |  |  |  |  |  |  |
| Pentane | 86.1 | 229.2 | $307 \cdot 0$ | $4 \cdot 87$ | $5 \cdot 86$ | $8 \cdot 12$ | 8.95 | $1 \cdot 20$ | $1 \cdot 67$ | 1.84 |
| Propyl acetate | 91-3 | 256.0 | 345-3 | $4 \cdot 67$ | 7.52 | $8 \cdot 42$ | $9 \cdot 31$ | 1.28 | 1.80 | 1.99 |
|  |  |  |  |  |  |  | Mean | $1 \cdot 32$ | 1.78 | 1.98 |

gases is small. This is shown by many of their properties, e.g., their low boiling points and critical temperatures; they also show a marked deviation from Sutherland's law at lower temperatures.

When the ratio $\sigma_{p} / \sigma_{\eta}$ is considered, it will be seen that hydrogen falls into line with the other substances. The values oscillate about a mean of 1.78 ; the deviations may be ascribed chiefly to the omission of a correction for the shape of the molecule, but in some cases, notably with chlorine, may be due to uncertainties in the viscosity data. It is evident that, by comparing molecular volumes at equal surface tensions, the effect of the intermolecular field is largely eliminated, and a closer parallelism with the dimensions of molecules calculated from gas viscosities is obtained.

The ratio $\sigma_{c} / \sigma_{\eta}$ calculated from the critical volume again shows high values for hydrogen and helium. For the remaining substances, the mean distance between the centres of adjacent molecules is about twice their diameter, i.e., the " edges" of such molecules are separated by about one diameter. For liquids at the ordinary temperature the molecules will approach one another still more closely and cannot therefore be treated as point centres of force. It is this fact which is the chief difficulty in the way of an adequate theory of capillarity.

In general, it is to be expected that zero volumes and critical volumes will not be capable of accurate analysis as an additive function, but will show deviations for those substances of which the molecules have peculiar intermolecular fields. On the other hand, parachors, in which the influence of this field is largely eliminated, should approach more closely to the requirements of an ideal additive function. As an extreme example, the constants found for elementary hydrogen and for hydrogen in organic compounds may be quoted-the zero volumes show a large difference but the parachors are nearly identical :

|  | $V_{0}$. | $[P]$. |
| :--- | :---: | :---: |
| $\mathrm{H}_{2}$ (element) $\ldots \ldots \ldots \ldots \ldots .$. | $22 \cdot 0$ | $35 \cdot 1$ |
| 2 H (combined) $\ldots \ldots \ldots \ldots \ldots$. | $13 \cdot 4$ | $34 \cdot 2$ |

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